The V-V Sector of the Lee Model

Herbert Rainer Petry and Mikolaj Sawicki *
Institut für Theoretische Kernphysik der Universität Bonn

Z. Naturforsch. **33a**, 1–6 (1978); received September 28, 1977

The Schrödinger-equation in the V-V sector of the Lee model is investigated. We obtain a three-body Lippmann-Schwinger equation, with a new type of disconnectedness, and construct the resolvent operator by means of a modified Faddeev technique.

1. Introduction

The Lee-model [1] provides us with a soluble field-theory, which allows a detailed discussion of renormalization effects. It exists in a variety of forms, the static model [1], the quasi-relativistic model [2] and the "Galilee" model [3, 4] which have in common the peculiar interaction $V \leftrightarrow N\theta$ between two kinds of fermions V, N and a boson θ, and differ otherwise in the specific choice of the free single-particle energies and the interaction form factor. In this paper we concentrate mainly on the so-called VV-sector which is the most difficult one considered so far [5]. As we will show in Sect. 2, it requires the solution of a Faddeev-type equation with renormalized energies. We assume in this paper that the coupling constant renormalization is always finite, which can be achieved, e.g., in the Galilee model or for specific form factors, but we allow for infinite energy renormalization.

Section II reviews the V and the $V\theta$-sector. In Sect. III we study the VV-sector, eliminate the unrenormalized quantities by renormalized ones and show, how the solutions of this sector can be obtained by the techniques developed by Faddeev [6].

2. The Lee-Model and its First Two Sectors

The Hamiltonian of the Lee-model is given by

$$H = H_0 + H_I\,,$$

where

$$H_0 = \int \mathrm{d}^3p\, [\omega_V(p)\, V^+(p)\, V(p) + \omega_N(p)\, N^+(p)\, N(p) + \varepsilon(p)\, a^+(p)\, a(p)]$$

and

$$H_I = \lambda_0 \int \mathrm{d}^3p\, \mathrm{d}^3q\, [V^+(p)\, N(p-q)\, a(q)\, \bar{h}(q) + N^+(p-q)\, a^+(p)\, V(p)\, h(q)]$$

with the form factor function

$$h\colon\ R^3 \to \mathbb{C}\,.$$

The notation is self-explanatory: $N^+(p)$ and $N(p)$ are the creation and annihilation operators for a fermion N of momentum p; $V^+(p)$, $V(p)$ and $a^+(p)$, $a(p)$ play the same role for the fermion V and the boson θ, respectively. The function $\omega_V(p)$ is the energy of the bare particle V of momentum p, and $\omega_N(p)$ and $\varepsilon(p)$ are the energies of the particles N and θ, respectively.

In our paper we consider two particular forms of Hamiltonians corresponding to two different situations:

(i) The case with relativistic kinematics, when $\omega_N(p) = (p^2 + m_N{}^2)^{1/2}$ etc. and a cut-off function is introduced in order to avoid diverences, and

(ii) The case with nonrelativistic particles, when $\omega_N(p) = p^2/2\,m_N$ etc., and no cut-off function is introduced, i.e. $h(p) = 1$.

Unless explicitly specified, our formulae hold in both cases. For simplicity, the particles are regarded as spinless, and the usual commutation rules hold:

$$[a(p), a^+(p')]_- = [V(p), V^+(p')]_+ = [N(p), N^+(p')]_+ = \delta(p-p')$$

and all other commutators (or anticommutators) vanish. The two operators Q_1, Q_2:

$$Q_1 = \int \mathrm{d}^3p\, [V^+(p)\, V(p) + N^+(p)\, N(p)]\,,$$
$$Q_2 = \int \mathrm{d}^3p\, [V^+(p)\, V(p) + a^+(p)\, a(p)]$$

are constants of motions and their eigenvalues define the particular "sectors" of the Lee-model.

II.1. The single V sector is defined by the eigenvalues $Q_1 = 1$ and $Q_2 = 1$, so that the general form

* Present address: Institute of Theoretical Physics, University of Warsaw.

Reprint requests to Dr. H. R. Petry, Institut für Theoretische Kernphysik der Universität Bonn, Nußallee 14–16, D-5300 Bonn.

of the state vector in this sector is

$$\psi_V = [\int d^3k\, \alpha(k)\, V^+(k) + \int d^3l\, d^3m\, \beta(l, m)\, N^+(l)\, a^+(m)]\, |0\rangle .$$

We require that the state ψ_V is an eigenstate of the total Hamiltonian H with eigenvalue E, i.e.

$$(E - H_0)\, \psi_V = H_I\, |\psi_V\rangle .$$

Comparing the coefficients of both components of the state vector ψ_V we obtain the equations

$$(E - \omega_V(k))\, \alpha(k) = \lambda_0 \int d^3q\, \bar{h}(q)\, \beta(k - q, q)$$

and

$$\beta(l, m) = \lambda_0\, h(m)\, [E - \omega_N(l) - \varepsilon(m)]^{-1} \cdot \alpha(l + m) .$$

They imply that

$$R(E, k)\, \alpha(k) = 0 \tag{0}$$

with

$$R(E, k) = E - \omega_V(k) - \lambda_0^2 \int d^3m\, \varrho(m) \cdot [E - \omega_N(k - m) - \varepsilon(m)]^{-1} \tag{1}$$

and $\varrho(m) = |h(m)|^2$.

A nontrivial solution of Eq. (0) exists if and only if $R(E, k)$ vanishes for some value of E denoted by $\omega_V{}^R(k)$, i.e. when

$$\omega_V{}^R(k) = \omega_V(k) + \lambda_0^2 \int d^3m\, \varrho(m) \cdot [\omega_V{}^R(k) - \omega_N(k - m) - \varepsilon(m)]^{-1} = \omega_V(k) + \delta\, \omega_V(k) .$$

The quantity $\omega_V{}^R$ has the meaning of a renormalized, i.e. physical, energy of the V-particle.

Evidently

$$\omega_V{}^R(k) - \omega_N(k - m) - \varepsilon(m) \neq 0$$

must hold (for all k, m), if the last formula has any meaning. If this condition is not satisfied, the V-particle will decay, a case which we exclude. We also exclude the possibility that the equation

$$R(E, k) = 0$$

has other solutions than $\omega_V{}^R(k)$.

Since $R(\omega_V{}^R(k), k) = 0$, we redefine

$$R(E, k) = R(E, k) - R(\omega_V{}^R(k), k) = (E - \omega_V{}^R(k)) \cdot Z(E, \omega_V{}^R(k), k) , \tag{2}$$

where

$$Z(E, E', k) = 1 + \lambda_0^2 \int d^3q\, \varrho(q) \cdot [E - \omega_N(k - q) - \varepsilon(q)]^{-1} \cdot [E' - \omega_N(k - q) - \varepsilon(q)]^{-1} .$$

There exist cases in which R is not well-defined but Z is. It turns out that $R(E, k)$ appears also in the other sectors of the Lee-model and contains the only divergent integrals of the theory which have to be renormalized. If one succeeds in showing that R can be consistently redefined by Eq. (2) in all the other equations of motion (belonging to different sectors), one has still a meaningful theory.

The function $Z(E, \omega_V{}^R(k), k)$ is connected with the normalization of the state with energy $\omega_V{}^R(k)$. Indeed, set

$$\alpha(k) = \delta(k - p)\, Z^{-1/2}(\omega_V{}^R(k), \omega_V{}^R(k), k) .$$

According to the previous discussion, this choice defines the states $|p\rangle$ with energy $\omega_V{}^R(p)$ by

$$|p\rangle = (V^+(p) + \lambda_0 \int d^3m\, h(m) \cdot [\omega_V{}^R(p) - \omega_N(p - m) - \varepsilon(m)]^{-1} \cdot N^+(p - m)\, a^+(m))\, |0\rangle \cdot Z^{-1/2}(\omega_V{}^R(p), \omega_V{}^R(p), p) .$$

One finds that

$$\langle p | p' \rangle = \delta(p - p') ,$$

i.e. the state $|p\rangle$ is correctly normalized as an eigenstate of energy and momentum.

II.2. The $V\theta$-sector is characterized by the eigenvalues $Q_1 = 1$ and $Q_2 = 2$, and the general form of the state vector is

$$\psi_{V\theta} = \int d^3k\, d^3m\, \alpha(k, m)\, V^+(k)\, a^+(m)\, |0\rangle + \int d^3k\, d^3m_1\, d^3m_2\, \beta(k, m_1, m_2) \cdot N^+(k)\, a^+(m_1)\, a^+(m_2)\, |0\rangle .$$

Bose-Einstein statistics requires that

$$\beta(k, m_1, m_2) = \beta(k, m_2, m_1) .$$

If $\psi_{V\theta}$ is an eigenstate of energy, it must hold that:

$$(E - H_0)\, \psi_{V\theta} = H_I\, \psi_{V\theta}$$

which yields

$$(E - \omega_V(k) - \varepsilon(m))\, \alpha(k, m) = 2\lambda_0 \int d^3q\, \bar{h}(q)\, \beta(k - q, q, m) ,$$

$$(E - \omega_N(k) - \varepsilon(m_1) - \varepsilon(m_2))\, \beta(k, m_1, m_2) = S_{m_1 m_2}\, \lambda_0\, h(m_1)\, \alpha(k + m_1, m_2) ,$$

where $S_{m_1 m_2}$ is the symmetrization operator in the boson momenta:

$$S_{m_1 m_2} f(m_1, m_2) := \tfrac{1}{2} (f(m_1, m_2) + f(m_2, m_1)) .$$

Eliminating β by means of the last formula leads to the equation

$$R(E-\varepsilon(m),k)\,\alpha(k,m)=\lambda_0^2\,h(m)\int d^3q\,\bar{h}(q)\cdot[E-\varepsilon(m)-\omega_N(k-q)-\varepsilon(q)]^{-1}\alpha(k-q+m,q).$$

Using formula (2), whih redefines R with help of the renormalized energy $\omega_V{}^R$ and introducing the function $\tilde{\alpha}(k,l)$ by

$$\tilde{\alpha}(k,l)=\alpha(k,l)/h(l)$$

we find immediately

$$(E-\omega_V{}^R(k)-\varepsilon(m))\cdot Z(\omega_V{}^R(k),E-\varepsilon(m),k)\,\tilde{\alpha}(k,m)=\lambda_0^2\int d^3q\,\varrho(q)\,[E-\varepsilon(m)-\omega_N(k-q)-\varepsilon(q)]^{-1}\cdot\tilde{\alpha}(k-q+m,q)\,.$$

This equation shows that the $V\theta$-sector is indeed fully described by renormalized quantities.

3. The VV-Sector

3.1. Renormalization

The VV-sector is characterized by the values $Q_1=2$ and $Q_2=2$; the general form of a state in this sector is

$$\begin{aligned}\psi_{VV}=&\int d^3k_1\,d^3k_2\,\alpha(k_1,k_2)\,V^+(k_1)\,V^+(k_2)\,|0\rangle\\&+\int d^3k\,d^3l\,d^3m\,\beta(k,l,m)\,V^+(k)\,N^+(l)\,a^+(m)\,|0\rangle\\&+\int d^3l_1\,d^3l_2\,d^3m_1\,d^3m_2\,\gamma(l_1,l_2,m_1,m_2)\,N^+(l_1)\,N^+(l_2)\,a^+(m_1)\,a^+(m_2)\,|0\rangle\end{aligned}$$

and the correct statistics requires that

$$\alpha(k_1,k_2)=-\alpha(k_2,k_1)\,,\qquad\gamma(l_1,l_2,m_1,m_2)=-\gamma(l_2,l_1,m_1,m_2)=\gamma(l_1,l_2,m_2,m_1)\,.$$

If ψ_{VV} is an eigenfunction of the Hamiltonian H with eigenvalue E, then

$$(E-H_0)\,\psi_{VV}=H_I\,\psi_{VV}\,,$$

which implies that

$$[E-\omega_V(k_1)-\omega_V(k_2)]\,\alpha(k_1,k_2)=A_{k_1k_2}\,\lambda_0\int d^3q\,\bar{h}(q)\,\beta(k_1,k_2-q,q)\,,$$

$$[E-\omega_V(k)-\omega_N(l)-\varepsilon(m)]\,\beta(k,l,m)=2\,\lambda_0\,h(m)\,\alpha(k,l+m)+4\,\lambda_0\int d^3q\,\bar{h}(q)\,\gamma(k-q,l,q,m)$$

and

$$[E-\omega_N(l_1)-\omega_N(l_2)-\varepsilon(m_1)-\varepsilon(m_2)]\,\gamma(l_1,l_2,m_1,m_2)=-A_{l_1l_2}\,S_{m_1m_2}\,\lambda_0\,h(m_2)\,\beta(l_2+m_2,l_1,m_1)$$

where $A_{l_1l_2}$ and $S_{m_1m_2}$ are the antisymmetrisation and symmetrisation operators in fermion and boson momenta, respectively,

$$A_{l_1l_2}f(l_1,l_2)=\tfrac{1}{2}\,(f(l_1,l_2)-f(l_2,l_1))$$

and $S_{m_1m_2}$ has been defined in the last section. We define $\tilde{\beta}$ by

$$\tilde{\beta}(k,l,m)=\beta(k,l,m)/h(m)$$

and eliminate γ. The result is given by the equations:

$$\begin{aligned}(E-\omega_V(k)-\omega_N(l)-\varepsilon(m))\,\tilde{\beta}(k,l,m)\\=2\,\lambda_0\,\alpha(k,l+m)+\lambda_0^2\int d^3q\,\varrho(q)\,[E-\omega_N(l)-\varepsilon(m)-\omega_N(k-q)-\varepsilon(q)]^{-1}\\\cdot[\tilde{\beta}(k-q+m,l,q)-\tilde{\beta}(l+m,k-q,q)+\tilde{\beta}(k,l,m)-\tilde{\beta}(l+q,k-q,m)]\end{aligned}\tag{3}$$

and

$$[E-\omega_V(k_1)-\omega_V(k_2)]\,\alpha(k_1,k_2)=\tfrac{1}{2}\,\lambda_0\int d^3q\,\varrho(q)\,[\tilde{\beta}(k_1,k_2-q,q)-\tilde{\beta}_2(k_2,k_1-q,q)]\,.\tag{4}$$

We observe that the third term under integral sign in (3) provides us, in an analogy to the $V\theta$-sector, with the off-shell energy renormalization of the V-particle energy $\omega_V(k)$; proceeding as in the previous section we rewrite Eq. (3) in the form

$$\begin{aligned}[E-\omega_V{}^R(k)-\omega_N(l)-\varepsilon(m)]\,Z(E-\omega_N(l)-\varepsilon(m),\omega_V{}^R(k),k)\,\tilde{\beta}(k,l,m)\\=2\,\lambda_0\,\alpha(k,l+m)+\lambda_0^2\int d^3q\,\varrho(q)\,[E-\omega_N(l)-\varepsilon(m)-\omega_N(k-q)-\varepsilon(q)]^{-1}\\\cdot[-\tilde{\beta}(l+q,k-q,m)+\tilde{\beta}(k-q+m,l,q)-\tilde{\beta}(l+m,k-q,q)]\end{aligned}\tag{5}$$

where $Z(E - \omega_N(l) - \varepsilon(m), \omega_V{}^R(k), k)$ is again the half-off-shell value of $Z(E, E', k)$ defined in the last section. Equation (5) is then expressed by renormalized quantities only. We try to do this for (4), too.

To this end we first rewrite the Eq. (5) in the form of a Lippmann-Schwinger equation; (hereafter we shall use the symbols k, l and m to describe the momenta of particles V, N and θ, respectively):

$$\int d^3k'\, d^3l'\, d^3m' \langle k, l, m | G_{03}^R(E)^{-1} | k', l', m'\rangle \tilde{\beta}(k', l', m') \\ = 2\lambda_0 \alpha(k, l+m) + \int d^3k'\, d^3l'\, d^3m' \langle k, l, m | V(E) | k', l', m'\rangle \tilde{\beta}(k', l', m'), \tag{6}$$

where

$$\langle k, l, m | G_{03}^R(E)^{-1} | k', l', m'\rangle \\ = \delta(P - P')\, \delta(l - l')\, \delta(m - m') [E - \omega_V{}^R(k) - \omega_N(l) - \varepsilon(m)] \cdot Z(E - \omega_N(l) - \varepsilon(m), \omega_V{}^R(k), k)$$

and P, P' are the total momenta of the system:

$$P = k + l + m, \qquad P' = k' + l' + m'.$$

The potential V has the form

$$V(z) = V_1(z) + V_2(z) + V_x(z)$$

with

$$\langle k, l, m | V_1(E) | k', l', m'\rangle \\ = -\delta(P - P')\, \delta(m - m')\, \lambda_0{}^2 \varrho(k - l') [E - \omega_N(l) - \varepsilon(m) - \omega_N(l') - \varepsilon(k - l')]^{-1},$$

$$\langle k, l, m | V_2(E) | k', l', m'\rangle \\ = +\delta(P - P')\, \delta(l - l')\, \lambda_0{}^2 \varrho(m') [E - \omega_N(l) - \varepsilon(m) - \omega_N(k - m') - \varepsilon(m')]^{-1},$$

and

$$\langle k, l, m | V_x(E) | k', l', m'\rangle \\ = -\delta(P - P')\, \delta(k - l' - m')\, \lambda_0{}^2 \varrho(m') [E - \omega_N(l) - \varepsilon(m) - \omega_N(k - m') - \varepsilon(m')]^{-1}.$$

The detailed discussion of the potentials is given in the next section. We define now the total Green operator G by

$$G(E) = [G_{03}^R(E)^{-1} - V(E)]^{-1},$$

and rewrite the Eq. (6) in the form

$$\tilde{\beta}(k, l, m) = \int d^3k', d^3l'\, d^3m' \langle k, l, m | G(E) | k', l', m'\rangle\, 2\lambda_0 \alpha(k', l' + m').$$

Substituting the last equation into (4) we get

$$[E - \omega_V(k_1) - \omega_V(k_2)]\, \alpha(k_1, k_2) \\ = \lambda_0{}^2 \int d^3q\, \varrho(q)\, d^3k'\, d^3l'\, d^3m' \langle k_1, k_2 - q, q | G(E) | k', l', m'\rangle\, \alpha(k', l' + m') - (k_1 \leftrightarrow k_2). \tag{7}$$

We introduce now the T-operator for the $VN\theta$-system:

$$T(z) = V(z) + V(z)\, G(z)\, V(z). \tag{8}$$

The following identities hold

$$G(z) = G_{03}^R(z) + G_{03}^R(z)\, T(z)\, G_{03}^R(z) \tag{9}$$

and

$$T(z) = V(z) + V(z)\, G_{03}^R(z)\, T(z). \tag{10}$$

We insert now (9) into Eq. (7) and observe that the first term of Eq. (9) provides us with the renormalization of the energies of the V-particles and that we can write

$$[E - \omega_V{}^R(k_1) - \omega_V{}^R(k_2) + \Delta E(k_1, k_2)]\, \alpha(k_1, k_2) = \lambda_0{}^2 \int d^3q\, \varrho(q)\, d^3k'\, d^3l'\, d^3m' \\ \cdot \langle k_1, k_2 - q, q | G_{03}^R(E) \cdot T(E) \cdot G_{03}^R(E) | k', l', m'\rangle \cdot \alpha(k', l' + m') - (k_1 \leftrightarrow k_2),$$

where

$$\Delta E(k_1, k_2) = \lambda_0^2 \int d^3m\, \varrho(m)\, \{[\omega_V{}^R(k_1) - \omega_N(k_2 - m) - \varepsilon(m)]^{-1} - [E - \omega_V{}^R(k_1) - \omega_N(k_2 - m) - \varepsilon(m)]^{-1} \cdot Z(E - \omega_N(k_2 - m) - \varepsilon(m), \omega_V{}^R(k_1), k_1)\} + (k_1 \leftrightarrow k_2)\,.$$

is a *finite* quantity. This achieves our goal to express (7) in terms of renormalized energies.

3.2. The Faddeev equations

We are now coming to the construction of the three-body operator, defined by Eq. (8); to this end we consider the potentials V_1, V_2 and V_x, defined in the last section. The potentials V_1 and V_2 correspond to the two-body energy-dependent interactions $V-N$ and $V-\theta$ imbedded in a three-body space, as is exhibited by the presence of delta functions conserving the momentum of spectator particles θ and N, respectively. Therefore, they give rise to the usual disconnected graphs in the three-body space, shown in Figures 1a, 1b. (The

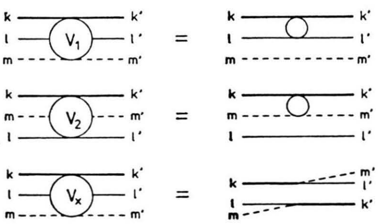

Fig. 1. Graphical representation of the disconnectedness in the potentials V_1, V_2 and V_x.

overall delta function, common for all three potentials represents, of course, conservation of the total momentum.) In contrast to V_1 and V_2, the interaction V_x changes all individual momenta of the particles, but due to the additional delta function gives still rise to the disconnected diagram 1c, yielding in this way a new type of a quasi-three-body force specific for the Lee-model.

It is interesting to note that the iterated product $V_x G_{03}^R V_x$ generates the conservation of momentum of the V-particle as it is shown in Fig. 2, and, therefore, corresponds to the two-body $N\theta$-interaction originally not present in the potential of our Lippmann-Schwinger equation.

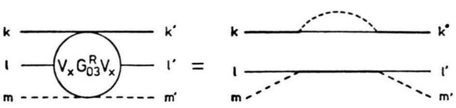

Fig. 2. The disconnectedness generated by the product $V_x G_{03}^R V_x$.

In order to construct T we have to sum up all diagrams exhibiting the same type of disconnectedness. To this end we consider all possible types of iterations in Equation (10). Following Stingl [7] we introduce a symbolic notation δ_N, δ_θ, δ_x and δ_V for the delta functions in V_1, V_2, V_x and $V_x G_{03}^R V_x$ respectively and write, for instance "$\delta_x \delta_x \to \delta_V$", i.e. two terms with momentum δ-functions δ_x and the operator G_{03}^R, sandwiched in between, give after operator multiplication a term with a δ-function δ_V. By inspection of all possible diagrams we find the following "disconnectedness multiplication table" (see Fig. 3) where the empty box means that

	δ_N	δ_θ	δ_x	δ_V
δ_N	δ_N			
δ_θ		δ_θ		
δ_x			δ_V	δ_x
δ_V			δ_x	δ_V

Fig. 3. The disconnectedness table.

the corresponding "disconnectedness multiplication" leads to a fully connected graph.

Now it is quite easy to sum up all graphs with a given type of disconnectedness: we define

$$t_1 = V_1 + V_1 G_{03}^R V_1 + V_1 G_{03}^R V_1 G_{03}^R V_1 + \cdots,$$
$$t_2 = V_2 + V_2 G_{03}^R V_2 + V_2 G_{03}^R V_2 G_{03}^R V_2 + \cdots,$$
$$t_x = V_x + V_x G_{03}^R V_x G_{03}^R V_x + \cdots,$$
$$t_V = V_x G_{03}^R V_x + V_x G_{03}^R V_x G_{03}^R V_x G_{03}^R V_x + \cdots,$$

and observe that the following integral equations hold:

$$t_1 = V_1 + V_1 G_{03}^R t_1\,,$$
$$t_2 = V_2 + V_2 G_{03}^R t_2\,,$$
$$\begin{pmatrix} t_x \\ t_V \end{pmatrix} = \begin{pmatrix} V_x \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & V_x \\ V_x & 0 \end{pmatrix} G_{03}^R \begin{pmatrix} t_x \\ t_V \end{pmatrix}.$$

We introduce now the channel Green operators G_α for the three potentials V_1, V_2 and V_x contributing to the full Green operator G,

$$G_\alpha(z) = (G_{03}^R(z)^{-1} - V_\alpha(z))^{-1}, \quad \alpha = 1, 2, x,$$

and observe that the following relations hold:

$$V_i\, G_i = t_i\, G_{03}^R, \quad i = 1, 2, \tag{12}$$

$$V_x G_x = (t_x + t_V)\, G_{03}^R\,. \tag{13}$$

Now we repeat Faddeev's procedure and split the total operator T into three parts

$$T = T_1 + T_2 + T_3$$

where

$$T_1 = V_1 + V_1 G V\,,$$
$$T_2 = V_2 + V_2 G V\,,$$
$$T_3 = V_x + V_x G V\,.$$

Using the resolvent identity

$$G = G_\alpha + G_\alpha \bar{V}_\alpha G\,, \quad \alpha = 1, 2, x\,,$$

where

$$\bar{V}_\alpha = V - V_\alpha$$

we obtain with help of (12) for $i = 1, 2$:

$$T_i = t_i + t_i G_{03}^R \sum_{\alpha \neq i} T_\alpha\,,$$

and, using Eq. (13)

$$T_3 = t_3 + t_3 G_{03}^R (T_1 + T_2)\,,$$

where we defined

$$t_3 = t_x + t_V\,.$$

We rewrite the last equations in the form

$$T_i = t_i + t_i G_{03}^R \sum_{j \neq i} T_j\,, \quad i = 1, 2, 3, \tag{14}$$

and observe that the kernel of our Faddeev equations (14) becomes connected after a single iteration so that (14) could be solved in the standard way. The solutions T_i have the form

$$T_i = t_i + W_i\,,$$

where W_i is the fully connected part of T_i.

Inserting the last formula in Eq. (11) yields

$$\begin{aligned}[E - \omega_V{}^R(k_1) - \omega_V{}^R(k_2) + \Delta E(k_1, k_2) + \Delta E_3(k_1, k_2)]\, \alpha(k_1, k_2) \\ = \lambda_0{}^2 \int \mathrm{d}^3 q\, \varrho(q)\, \mathrm{d}^3 k'\, \mathrm{d}^3 l'\, \mathrm{d}^3 m' \langle k_1, k_2 - q, q |\, G_{03}^R(E)\, [T_1(E) + T_2(E) + W_3(E)] \\ \cdot G_{03}^R(E)\, | k', l', m' \rangle\, \alpha(k, l', m') - (k_1 \leftrightarrow k_2)\,.\end{aligned} \tag{15}$$

The additional finite energy shift $\Delta E_3(k_1, k_2)$ is calculated in Appendix A.

The homogeneous Eq. (15) corresponds to an effective two-body problem. Its solutions can be calculated, at least numerically, by standard methods. In the next step Eq. (6) is solved for $\tilde{\beta}(k, l, m)$. Knowing $\beta(k, l, m)$ we obtain immediately the function $\gamma(l_1, l_2, m_1, m_2)$.

Appendix A

We demonstrate now that the term $G_{03}^R\, t_3\, G_{03}^R$ produces an additional energy shift $\Delta E_3(k_1, k_2)$. To this end we calculate

$$\begin{aligned} I = \int & \mathrm{d}^3 q\, \varrho(q)\, \mathrm{d}^3 k'\, \mathrm{d}^3 l'\, \mathrm{d}^3 m'\, \mathrm{d}^3 k''\, \mathrm{d}^3 l''\, \mathrm{d}^3 m''\, \mathrm{d}^3 k'''\, \mathrm{d}^3 l'''\, \mathrm{d}^3 m''' \\ & \cdot \langle k_1, k_2 - q, q |\, G_{03}^R\, | k'', l'', m'' \rangle \langle k'', l'', m'' |\, t_x + t_V\, | k''', l''', m''' \rangle \\ & \cdot \langle k''', l''', m''' |\, G_{03}^R\, | k', l', m' \rangle\, \alpha(k', l' + m') - (k_1 \leftrightarrow k_2)\,. \end{aligned} \tag{A1}$$

We introduce the following notation

$$\begin{aligned} \langle k, l, m |\, G_{03}^R\, | k', l', m' \rangle &= \delta(k - k')\, \delta(l - l')\, \delta(m - m')\, g_0(E, k', l', m')\,, \\ \langle k, l, m |\, t_x(E)\, | k', l', m' \rangle &= \delta(k + l + m - k' - l' - m')\, \delta(k - l' - m')\, \tau_x(E, k, l, m, m')\,, \\ \langle k, l, m |\, t_V(E)\, | k', l', m' \rangle &= \delta(k + l + m - k' - l' - m')\, \delta(k - k')\, \tau_V(E, k, l, m, m')\,. \end{aligned} \tag{A2}$$

Inserting (A2) into (A1) and performing the integration over all delta functions we obtain

$$I = -\Delta E_3(k_1, k_2)\, \alpha(k_1, k_2)$$

with

$$\begin{aligned} \Delta E_3(k_1, k_2) = & \int \mathrm{d}^3 q\, \varrho(q)\, \mathrm{d}^3 l'\, g_0(E, k_1, k_2 - q, q)\, \tau_x(E, k_1, k_2 - q, q, k_1 - l')\, g_0(E, k_2, l', k_1 - l') \\ & - \int \mathrm{d}^3 q\, \varrho(q)\, \mathrm{d}^3 l'\, g_0(E, k_1, k_2 - q, q)\, \tau_V(E, k_1, k_2 - q, q, k_2 - l') \\ & \cdot g_0(E, k_1, l', k_2 - l) + (k_1 \leftrightarrow k_2)\,. \end{aligned}$$

[1] T. D. Lee, Phys. Rev. **95**, 1329 (1954).
[2] S. S. Schweber, An Introduction to Relativistic Quantum Field Theory, chapter 12. Row, Peterson & Company, Evanston 1961.
[3] J. M. Lévy-Leblond, Commun. Math. Phys. **4**, 157 (1967).
[4] R. Schrader, Commun. Math. Phys. **10**, 155 (1968).
[5] L. M. Scarfone, J. Math. Phys. **9**, 246 (1968).
[6] L. D. Faddeev, JETP **12**, 1014 (1961).
[7] M. Stingl and A. T. Stelbovics, Universität Münster preprint, 1977 (unpublished).